AL-FAID MEMORIAL INSTITUTE
WATERHAIL, BUDGAM

CLASS X

MATHEMATICS

CHAPTER: PROBABILITY

Concepts $\longrightarrow$ Examples $\longrightarrow$ Applications

From the Basics to Class X

These notes begin with the elementary ideas of probability developed in earlier classes and gradually build towards the Class X classical definition of probability.

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Contents

1. Introduction to Probability

In our daily life, we often make statements such as:

- "It may rain tomorrow."
- "There is a chance that our team will win."
- "The coin may show Head."
- "There is a possibility of getting a six when a die is thrown."

All these statements involve some degree of **uncertainty**.

Mathematics provides a systematic way to measure this uncertainty. This branch of Mathematics is called **Probability**.

Probability is the mathematical measure of the likelihood or chance of occurrence of an event.

The word "probability" is closely related to words such as:

$$\boxed{\text{chance, possibility, likelihood}}$$

1.1 Why do we Study Probability?

Probability is useful whenever the result of an activity cannot be known with certainty in advance.

For example:

- Tossing a coin.
- Throwing a die.
- Drawing a card from a deck.
- Selecting a coloured ball from a bag.
- Predicting the possibility of rain.
- Studying games and sports.

Example

Suppose a fair coin is tossed.
We cannot say with certainty whether the result will be Head or Tail before tossing it.
However, we can mathematically describe the chance of obtaining Head.

This is where Probability becomes useful.

2. Probability: Foundation from Earlier Classes

Before studying the Class X definition, let us revise the basic ideas that form the foundation of Probability.

The most important terms are:

Term	**Meaning**
Experiment	An activity whose result is observed
Outcome	A possible result of an experiment
Sample Space	Collection of all possible outcomes
Event	One or more specified outcomes
Equally Likely Outcomes	Outcomes having equal chances
Probability	Numerical measure of chance

3. Random Experiment

An experiment is called a **Random Experiment** when:

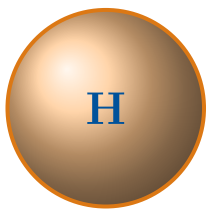
1. all possible outcomes are known in advance, but
2. the exact outcome cannot be predicted with certainty before the experiment is performed.

Example

Consider tossing a coin.
The possible results are:

$$H, \quad T.$$

We know these possible results beforehand, but we cannot know with certainty which one will occur before tossing the coin.
Therefore, tossing a coin is a random experiment.



One possible outcome: Head

3.1 Non-Random or Certain Activity

An activity whose result can be predicted with certainty is not considered a random experiment.

Example

The Sun rising in the east is a predictable natural occurrence. There is no uncertainty about the expected direction.
Thus, it is not an appropriate example of a random experiment.

4. Outcome

An **Outcome** is a possible result of a random experiment.

Example

When a die is thrown, the possible outcomes are

$$1,\ 2,\ 3,\ 4,\ 5,\ 6.$$

Each of these numbers is an outcome.

$$\boxed{\text{Outcome} = \text{A possible result of an experiment}}$$

5. Sample Space

The **Sample Space** is the collection of all possible outcomes of a random experiment.

It is commonly denoted by

$$\boxed{S}$$

Example

For one toss of a coin,

$$S = \{H, T\}.$$

For one throw of a die,

$$S = \{1, 2, 3, 4, 5, 6\}.$$

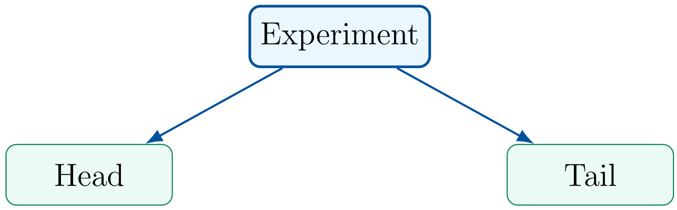


Fig. 1 : Sample Space of a Coin Toss

6. Event

An **Event** is a collection of one or more outcomes of a random experiment that satisfies a particular condition.

Example

When a die is thrown, consider the event:

$$E = \text{getting an even number.}$$

The even outcomes are

$$2,\ 4,\ 6.$$

Therefore,

$$E = \{2, 4, 6\}.$$

An event may contain:

- one outcome,
- more than one outcome,
- or, in some cases, all possible outcomes.

7. Simple Event

An event containing only one outcome is called a **Simple Event**.

Example

When a die is thrown,

$$E = \text{getting a } 5$$

has only one favourable outcome:

$$E = \{5\}.$$

Hence, it is a simple event.

8. Compound Event

An event containing more than one outcome is called a **Compound Event**.

Example

When a die is thrown,

$$E = \text{getting an odd number}.$$

Then

$$E = \{1, 3, 5\}.$$

Since the event contains three outcomes, it is a compound event.

9. Equally Likely Outcomes

Two or more outcomes are said to be **Equally Likely** if they have the same chance of occurring.

Example

For a fair coin,

$$H$$

and

$$T$$

are equally likely outcomes.
Similarly, for a fair die, the six faces

$$1, 2, 3, 4, 5, 6$$

are equally likely.

Common Mistake

Important:
"Possible outcomes" does not automatically mean "equally likely outcomes."
For example, the outcome of a basketball shot may be "score" or "miss", but these two outcomes need not have equal chances.

10. Fair and Unfair Experiments

A random experiment is called **Fair** when all its possible outcomes are equally likely.

Example

A properly balanced coin is considered a fair coin because

$$P(H) = P(T).$$

Similarly, a properly balanced die is considered a fair die because each face has the same chance of appearing.

11. Experimental or Empirical Probability

Before reaching the Class X theoretical definition, it is useful to understand how probability can be estimated through actual experiments.

Suppose an experiment is repeated a large number of times.

The ratio

$$\frac{\text{Number of times the event occurs}}{\text{Total number of trials}}$$

gives the **Experimental Probability** or **Empirical Probability**.

Thus,

$$\boxed{P(E) \approx \frac{\text{Number of times event } E \text{ occurs}}{\text{Total number of trials}}}$$

The symbol "≈" is used because the experimental value may not be exactly equal to the theoretical probability.

Example

A coin is tossed 100 times and Head occurs 57 times.
Then the experimental probability of getting Head is

$$P(H) \approx \frac{57}{100} = 0.57.$$

Number of Tosses	**Heads**	**Experimental Probability**
10	6	$\frac{6}{10} = 0.60$
20	11	$\frac{11}{20} = 0.55$
50	27	$\frac{27}{50} = 0.54$
100	52	$\frac{52}{100} = 0.52$

Student Tip

As the number of trials becomes large, the experimental probability generally tends to get closer to the theoretical probability, provided the experiment is conducted fairly and under similar conditions.

12. Theoretical or Classical Probability

We now move towards the central idea required at the Class X level.

When all possible outcomes of an experiment are **equally likely**, the probability of an event is determined by comparing the number of favourable outcomes with the total number of equally likely outcomes.

Classical Definition of Probability
If an event E has $n(E)$ favourable outcomes out of a total of $n(S)$ equally likely outcomes in the sample space S, then

$$\boxed{P(E) = \frac{n(E)}{n(S)}}$$

provided that

$$n(S) \neq 0.$$

12.1 Understanding the Formula

In

$$P(E) = \frac{n(E)}{n(S)},$$

- $P(E)$ = probability of event E,
- $n(E)$ = number of favourable outcomes,
- $n(S)$ = total number of equally likely outcomes in the sample space.

Student Tip

Easy way to remember:

$$\boxed{\text{Probability} = \frac{\text{Favourable Outcomes}}{\text{Total Possible Outcomes}}}$$

13. Probability of a Coin Toss

For a fair coin,

$$S = \{H, T\}.$$

Therefore,

$$n(S) = 2.$$

Let

$$E = \text{getting Head}.$$

Then

$$E = \{H\}$$

and

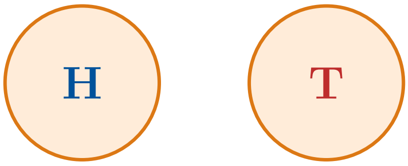
$$n(E) = 1.$$

Therefore,

$$P(E) = \frac{1}{2}.$$

Hence,

$$\boxed{P(H) = \frac{1}{2}}.$$



$S = \{H, T\}$

14. Probability Using a Die

A standard die has six faces numbered

$$1, 2, 3, 4, 5, 6.$$

Therefore,

$$S = \{1, 2, 3, 4, 5, 6\}$$

and

$$n(S) = 6.$$

Example

Find the probability of getting an even number.
The favourable outcomes are

$$E = \{2, 4, 6\}.$$

Thus,

$$n(E) = 3.$$

Therefore,

$$P(E) = \frac{3}{6} = \frac{1}{2}.$$

Hence,

$$\boxed{P(E) = \frac{1}{2}}.$$

1 **2** **3** **4**

Some possible faces of a die

15. Probability Using Playing Cards

A standard deck contains

$$52$$

cards.

It consists of four suits:

$$\text{Hearts, Diamonds, Clubs, Spades.}$$

Each suit contains

$$13$$

cards.

Thus,

$$4 \times 13 = 52.$$

Example

Find the probability of drawing a King from a well-shuffled deck.
There are four Kings in the deck.
Therefore,

$$n(E) = 4$$

and

$$n(S) = 52.$$

Hence,

$$P(E) = \frac{4}{52} = \frac{1}{13}.$$

Therefore,

$$\boxed{P(\text{King}) = \frac{1}{13}}.$$

16. Probability Using Objects in a Bag

Example

A bag contains 5 red balls and 3 blue balls.
Find the probability of drawing a blue ball at random.
Total number of balls:

$$5 + 3 = 8.$$

Number of blue balls:

$$3.$$

Therefore,

$$P(\text{Blue}) = \frac{3}{8}.$$

Hence,

$$\boxed{P(\text{Blue}) = \frac{3}{8}}.$$

17. Impossible Event

An event that **cannot occur** is called an **Impossible Event**.

Its probability is

$$\boxed{0}.$$

Example

A standard die has faces numbered from 1 to 6.
The event

$$E = \text{getting } 8$$

is impossible.
There are no favourable outcomes.
Thus,

$$P(E) = \frac{0}{6} = 0.$$

18. Sure or Certain Event

An event that **must occur** is called a **Sure Event** or **Certain Event**.

Its probability is

$$\boxed{1}.$$

Example

When a standard die is thrown, the event

$$E = \text{getting a number less than } 7$$

is certain.
All six outcomes satisfy this condition.
Therefore,

$$P(E) = \frac{6}{6} = 1.$$

19. Range of Probability

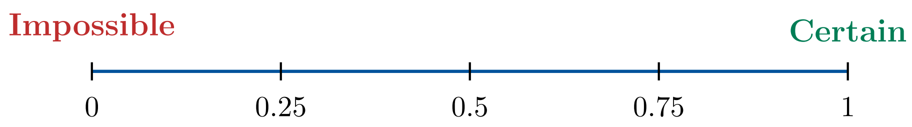
The probability of any event E always lies between 0 and 1.

Therefore,

$$\boxed{0 \leq P(E) \leq 1}$$

This gives us an important test.

- $P(E) = 0 \rightarrow$ impossible event.
- $0 < P(E) < 1 \rightarrow$ event has some uncertainty.
- $P(E) = 1 \rightarrow$ sure event.



20. Complementary Event

If E is an event, then the event that E does **not** occur is called the **Complement of** E.

It is commonly written as

$$\boxed{\text{not } E}$$

or

$$\boxed{E'}$$

depending on the notation being used.

The event E and its complement together cover all possible outcomes.
Therefore,

$$\boxed{P(E) + P(\text{not } E) = 1}$$

and hence,

$$\boxed{P(\text{not } E) = 1 - P(E)}$$

Example

If

$$P(E) = 0.35,$$

find the probability of "not E".
Using

$$P(\text{not } E) = 1 - P(E),$$

we get

$$P(\text{not } E) = 1 - 0.35 = 0.65.$$

Therefore,

$$\boxed{P(\text{not } E) = 0.65}.$$

Example

A die is thrown once.
Find the probability of **not getting a 6**.
Let

$$E = \text{getting a 6}.$$

Then

$$P(E) = \frac{1}{6}.$$

Therefore,

$$P(\text{not } E) = 1 - \frac{1}{6} = \frac{5}{6}.$$

Hence,

$$\boxed{\frac{5}{6}}.$$

21. Important Properties of Probability

For any event E:

1. $$\boxed{0 \le P(E) \le 1}$$

2. For an impossible event,

$$\boxed{P(E) = 0}$$

3. For a sure event,

$$\boxed{P(E) = 1}$$

4. For the complement,

$$\boxed{P(E) + P(\text{not } E) = 1}$$

5. Therefore,

$$\boxed{P(\text{not } E) = 1 - P(E)}$$

22. How to Solve a Probability Problem

Whenever you encounter a probability problem, follow these steps.

Step 1: Identify the random experiment.

Step 2: Write the sample space, if required.

Step 3: Identify the favourable outcomes.

Step 4: Count the total number of equally likely outcomes.

Step 5: Apply

$$P(E) = \frac{n(E)}{n(S)}.$$

Step 6: Simplify the answer.

23. Additional Solved Examples

Example

Example 1
A die is thrown once. Find the probability of getting a number greater than 4.
Solution
Sample space:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Favourable outcomes:

$$E = \{5, 6\}.$$

Therefore,

$$P(E) = \frac{2}{6} = \frac{1}{3}.$$

$$\boxed{\frac{1}{3}}$$

Example

Example 2

A card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting a red card.

There are 26 red cards.

Therefore,

$$P(\text{Red}) = \frac{26}{52} = \frac{1}{2}.$$

Hence,

$$\boxed{\frac{1}{2}}.$$

Example

Example 3

A bag contains 7 white balls and 5 black balls. One ball is drawn at random. Find the probability of getting a white ball.

Total balls:

$$7 + 5 = 12.$$

Favourable outcomes:

$$7.$$

Therefore,

$$P(\text{White}) = \frac{7}{12}.$$

$$\boxed{\frac{7}{12}}$$

Example

Example 4

A die is thrown once. Find the probability of getting a number which is not a multiple of 3.

Multiples of 3 are

$$3,\ 6.$$

Thus, the probability of getting a multiple of 3 is

$$\frac{2}{6} = \frac{1}{3}.$$

Therefore,

$$P(\text{not a multiple of 3}) = 1 - \frac{1}{3} = \frac{2}{3}.$$

Hence,

$$\boxed{\frac{2}{3}}.$$

24. Two Coin Tosses

When two coins are tossed simultaneously, the possible outcomes are:

$$HH,\ HT,\ TH,\ TT.$$

Thus,

$$S = \{HH, HT, TH, TT\}.$$

Therefore,

$$n(S) = 4.$$

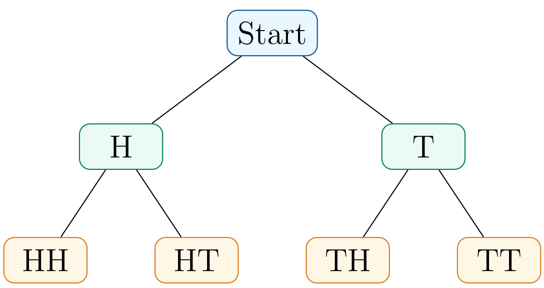


Fig. 2 : Outcomes when two coins are tossed

Example

Find the probability of getting exactly one Head.
The favourable outcomes are

$$HT,\ TH.$$

Therefore,

$$n(E) = 2.$$

Hence,

$$P(E) = \frac{2}{4} = \frac{1}{2}.$$

Therefore,

$$\boxed{\frac{1}{2}}.$$

25. Two Dice: Basic Idea

When two dice are thrown, each die has six possible outcomes.

Therefore, the total number of ordered outcomes is

$$6 \times 6 = 36.$$

For example,

$$(1,1),\ (1,2),\ (1,3),\ldots,(6,6).$$

Student Tip

When more than one object is involved, carefully count the **ordered outcomes** whenever the order matters.
For two dice,

$$(2,5)$$

and

$$(5,2)$$

are different outcomes.

Example

Find the probability that the sum of the numbers appearing on two dice is 7.
The favourable ordered pairs are

$$(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).$$

Thus,

$$n(E) = 6$$

and

$$n(S) = 36.$$

Therefore,

$$P(E) = \frac{6}{36} = \frac{1}{6}.$$

Hence,

$$\boxed{\frac{1}{6}}.$$

26. Transition to Class X Probability

By now, we have developed the basic probability ideas:

- Random experiment
- Outcome
- Sample space
- Event
- Equally likely outcomes
- Experimental probability
- Theoretical probability
- Impossible event
- Sure event
- Complementary event

The central Class X idea can now be stated very simply:

When the outcomes are equally likely,

$$\boxed{P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of equally likely outcomes}}}$$

This is the main principle that students should apply while solving the elementary probability problems at this level.

27. Experimental Probability vs Theoretical Probability

Experimental Probability	**Theoretical Probability**
Obtained from actual trials or observations.	Obtained mathematically from possible equally likely outcomes.
Depends upon observed results.	Depends upon the structure of the experiment.
$\frac{\text{Observed frequency}}{\text{Total trials}}$	$\frac{\text{Favourable outcomes}}{\text{Total equally likely outcomes}}$
May vary from one set of trials to another.	Remains fixed for the same ideal experiment.

Common Mistake

Do not apply

$$P(E) = \frac{n(E)}{n(S)}$$

blindly.
First check whether the outcomes being counted are **equally likely**.

28. Probability Formula Sheet

IMPORTANT FORMULAE

Classical Probability:

$$\boxed{P(E) = \frac{n(E)}{n(S)}}$$

Experimental Probability:

$$\boxed{P(E) \approx \frac{\textbf{Number of times } E \textbf{ occurs}}{\textbf{Total number of trials}}}$$

Complement:

$$\boxed{P(\textbf{not } E) = 1 - P(E)}$$

Therefore,

$$\boxed{P(E) + P(\textbf{not } E) = 1}$$

Range:

$$\boxed{0 \leq P(E) \leq 1}$$

Impossible Event:

$$\boxed{P(E) = 0}$$

Sure Event:

$$\boxed{P(E) = 1}$$

29. Common Mistakes to Avoid

Common Mistake

1. Do not confuse an **outcome** with an **event**.
2. Do not forget to count all possible outcomes.
3. Do not count favourable outcomes incorrectly.
4. Do not use the classical formula unless the outcomes are equally likely.
5. Always simplify the final fraction.
6. Probability can never be negative.
7. Probability can never be greater than 1.
8. When using a complement, remember:

$$P(\text{not } E) = 1 - P(E).$$

30. Quick Revision

Concept	**Remember**
Random Experiment	Result cannot be predicted with certainty beforehand.
Outcome	A possible result.
Sample Space	Set/collection of all possible outcomes.
Event	One or more specified outcomes.
Equally Likely	Same chance of occurring.
Impossible Event	Probability $= 0$.
Sure Event	Probability $= 1$.
Classical Probability	$P(E) = \dfrac{n(E)}{n(S)}$
Complement	$P(\text{not } E) = 1 - P(E)$
Range	$0 \leq P(E) \leq 1$

Final Thought

Probability does not tell us exactly what will happen.
It tells us how likely an outcome is to happen.

Understand the outcomes first; calculate the probability second.

— End of Basic Probability Notes —

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